TECHNISCHE UNIVERSITÄT
KAISERSLAUTERN

Department of Computer Science
AG Algorithms and Complexity

Issue Date: 01.06.2015
Version: 2015-05-29 15:14

7th Exercise sheet for Advanced Algorithmics, SS 15

Hand In: *Until Monday, 08.06.2015, 12:00am, in lecture, exercise sessions, hand-in box in stairwell 48-6 or via email.*

Problem 17

Define ZPP the class of decision problems that can be decided by polynomial-expected-runtime Las Vegas algorithms. Define RP, BPP and PP similarly for OSE-MC, TSE-MC and UE-MC algorithms, respectively.

Show (at least) three of the following statements:

a) $\mathcal{P} \subseteq \text{ZPP} \subseteq \text{RP} \subseteq \text{BPP} \subseteq \text{PP}$

b) $\text{RP} \subseteq \mathcal{NP}$

c) $\text{ZPP} = \text{RP} \cap \text{co-RP}$

d) $\mathcal{NP} \subseteq \text{co-RP} \implies \mathcal{NP} = \text{ZPP}$

e) $\mathcal{NP} \cup \text{co-}\mathcal{NP} \subseteq \text{PP}$

Problem 18

i) Argue why the definitions of “randomized δ-approximation algorithm” and “randomized δ-expected approximation algorithm” are not equivalent.

ii) Can you give an algorithm for a natural problem that is one, but not the other?

iii) Can you give a problem that can be solved with one, but not the other kind?

Problem 19

Consider the following problem P:

Input: Digraph $G = (V, E)$.

Solutions: Acyclic spanning subgraph $G' = (V, E')$ of G.

Goal: Maximise $|E'|$.

And furthermore the algorithm A:

1. Order V randomly.
2. Select as E'
   - all forward edges (w. r. t. the order from 1.) with probability $\frac{1}{2}$ and
   - all backwards edges otherwise.

Show that A is a randomized 2-expected approximation for P.

Problem 20

Consider the Weighted Vertex Cover problem, that is:

Input: A graph $G = (V, E)$ with vertex weights $w : V \to \mathbb{N}$.

Solutions: Sets of nodes $C \subseteq V$ so that every edge is covered, i. e. $u \in C$ or $v \in C$ for all $\{u, v, \} \in E$.

Goal: Minimise cover cost $w(C) = \sum_{v \in C} w(v)$.

And the algorithm A:

```
C = ∅
while E ≠ ∅ {
  select e = {v,t} ∈ E
  randomly choose x ∈ {v,t} with Pr[x = v] = w(t) / (w(v) + w(t))
  C = C ∪ {x}
  E = E \ {e | e incident of x}
}
return C
```

Show that A is a randomized 2-expected approximation algorithm for Weighted Vertex Cover.