TECHNISCHE UNIVERSITÄT
KAISERSLAUTERN

Department of Computer Science
AG Algorithms and Complexity

Issue Date: 08.06.2015
Version: 2015-06-08 18:14

8th Exercise sheet for Advanced Algorithmics, SS 15

Hand In: *Until Wednesday, 17.06.2015, 12:00am, in lecture, exercise sessions, hand-in box in stairwell 48-6 or via email.*

Problem 21

Let S a set of keys with $|S| = n$. Show that the expected height of a treap for S is logarithmic, i. e.

$$\mathbb{E}[\text{height}(n)] \in \mathcal{O}(\log n) \ .$$

Hint: You can approach this as follows.

1. Show that the $\text{depth}(x) \in \mathcal{O}(\log n)$ with high probability, for all elements $x \in S$.
2. Show that $\text{height}(n) \in \mathcal{O}(\log n)$ with high probability.
3. Derive the claim.

You will have to do some literature research. To get you started, here are some pointers.

- The value of Game A is also called (the expected number of) “left-to-right maxima” (in random permutations). Under this name it has been analysed deeply, e. g. in *The Art of Computer Programming Vol. 1* by Donald Knuth.
- The phrase “with high probability” has no clear definition. It often refers to bounds on the *tails* of a distribution. Such can be obtained by Markov or Chebyshev inequalities, others are called Chernoff bounds. The latter are the strongest; you can find details for instance in *Elements of Information Theory* by T. M. Cover and J. A. Thomas.
- You can think about the connection between treaps, regular binary search trees and Quicksort with respect to this analysis.

Problem 22

Show that the following statements hold for treaps of size n:

a) The expected runtime of operations INSERT and DELETE is $\mathcal{O}(\log n)$, respectively.

b) The expected number of rotations during an INSERT or DELETE operation is bounded above by two.

Problem 23

Devise an algorithm that splits treaps S as efficiently as possible into two treaps $S_{\leq k}$ and $S_{>k}$ which contain all keys in S that are $\leq k$ resp. $> k$.

Argue why your algorithm is correct and what runtime it has.